

Identification of Material Properties: Damping Ratio and Elastic Modulus

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Encap Motors

Abstract

The dynamic systems have the specific characteristics for some inputs and outputs. If the system is regarded as a linear model, there is a certain relationship between input and output. When some inputs are given for some dynamic systems, then one can always find the outputs for the inputs. In general, the relationships are obtained as the compliance(Displacement/Force), the mobility(Velocity/Force) or the acceleration(Acceleration/Force) in frequency domain. Given the one of the relationships, the dynamic properties of the systems can be identified by some mathematical procedure called an experimental modal analysis. In this report, the mobility as the response of the test materials was obtained by ratio of the velocity to the force. The damping ratio and elastic modulus were identified by it.

1 Frequency Response Function

To identify the dynamic characteristics, a transfer function is used generally as the method of frequency domain analysis. From the test, the force and the velocity are measured as the input and the output. So the transfer function in the frequency domain is defined by "mobility" in this case.

One-degree system

$$\begin{aligned} H_i(\beta_i) &= \frac{V}{F} = \frac{\text{Velocity}}{\text{Force}} \\ &= \frac{j\beta_i\Omega_i}{1 - \beta_i^2 + 2j\zeta_i\beta_i} \end{aligned} \quad (1)$$

where

$$\beta_i = \frac{\omega}{\Omega_i}; \quad \Omega_i = \text{natural frequency at } i\text{th mode}; \quad \omega = \text{frequency}[\text{rad/sec}]$$

Mode Decomposition

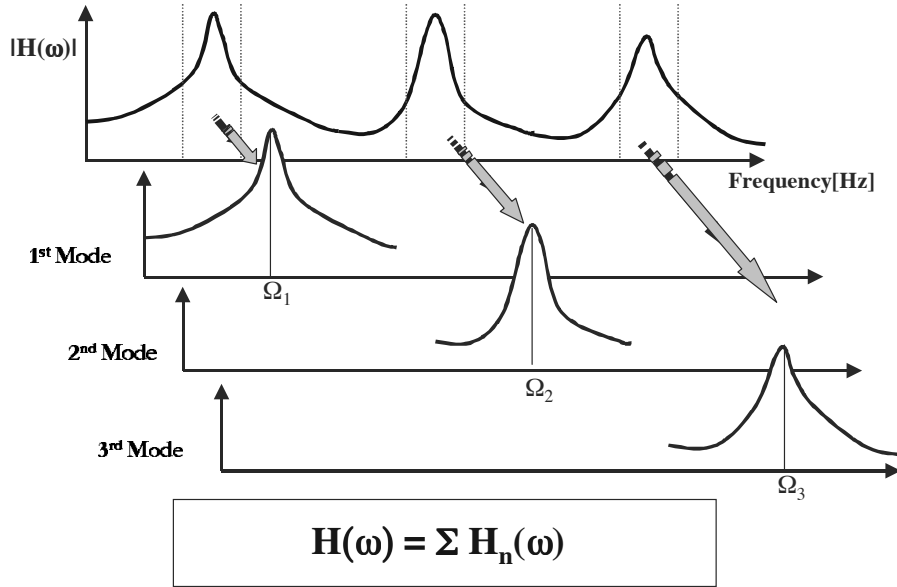


Figure 1: Decomposition of Multi-degree system

N-degree system

Under the linear assumption, N-degree system is considered as sum of the one-degree mode represented by (1).

$$H(\omega) = \sum_{i=1}^N H_i(\omega) = \sum_{i=1}^N H_i(\beta_i) \quad (2)$$

It is very useful but one should remind that the modes of the N-degree dynamic system must be enough away each other so that their modes cannot make interference with themselves. In this test case, each mode shows sufficient distance by itself and it can be considered as one degree model for each mode. One can divide the multi-degree system into one-degree system for each mode. So it is shown in Figure(1). Each mode can be regarded as one degree model.

2 Experiment Setup

Figure(2) illustrates the experimental setup. The major instruments are the following:

- Exciter(Shaker) : is used in order to excite the force into the system with a specific input such as white noise.
- Laser Vibrometer : picks up a velocity signal as a system response.

Experimental Setup

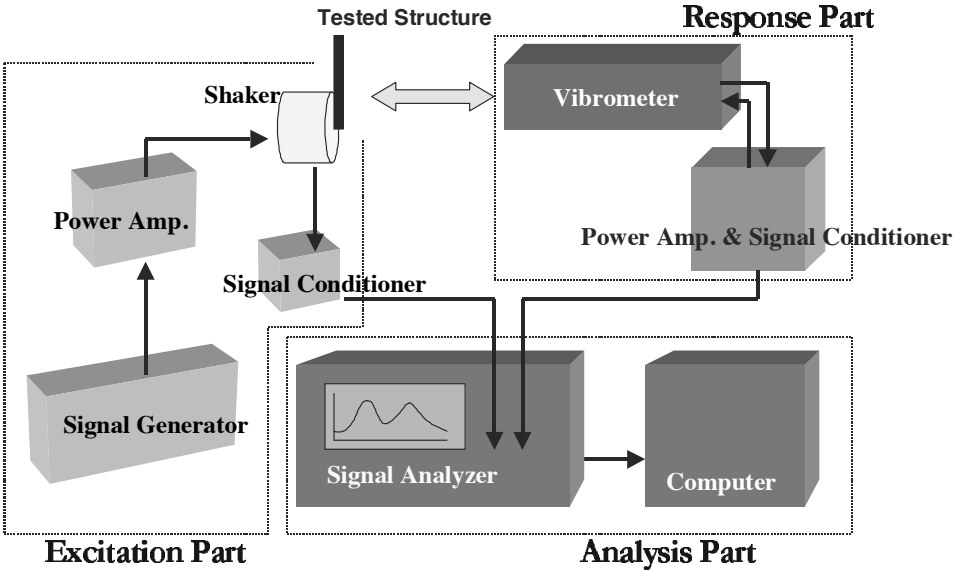


Figure 2: Experiment setup

- FFT Analyzer : makes and displays the frequency response function as the mobility by measurements of the force and the velocity.
- PC with GPIB interface : stores the mobility data via GPIB interface card and analyzes them to get material properties.

One decides one of input signal waveforms by using signal generator. In general, the input is chosen by white noise to keep constant power along all frequency range-actually for some considerable frequency range. Or sine sweep wave form could be. The FFT analyzer gives the FRF as a transfer function. In this test, the FRF(mobility) is directly used to identify the dynamic properties for simplicity. In order to use one degree formula, multi-degree response should be divided into many one degree model as shown in Figure(1). However using more deeper system identification scheme one could acquire input and output separatively and obtain the transfer function to use curve fit modal analysis. The transfer function gives us more detail information about the system.

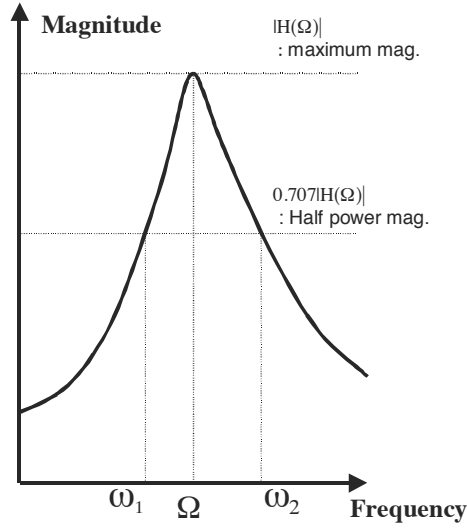


Figure 3: Determine damping ratio using magnitude around resonance

3 Calculation of Elastic modulus and Damping ratio

3.1 Damping Ratio

There are two ways to find the damping ratio using the frequency response function (FRF) : one is to use the magnitude of FRF around the resonance frequency, and the other is to use the phase slope around the resonance.

The first method:

$$\zeta_i = \frac{\omega_{i2} - \omega_{i1}}{2\Omega_i} \quad (3)$$

where ω_{i1} and ω_{i2} are the frequencies with half power magnitude of the maximum magnitude at Ω_i as shown in Figure(3).

The second method:

$$\zeta_i = \frac{-1}{\Omega_i \frac{d\phi}{d\omega}} \quad (4)$$

In this test, (3) is used but in case the modes are not clear, (4) may be useful.

3.2 Elastic Modulus

Natural Frequency and Young's Modulus

The materials tested can be modelled as a cantilever beam, and the relation between natural frequency and Young's modulus is given by the following formula.

To begin with, equation of motion of the cantilever beam is written mathematically,

$$\frac{d^4 w}{dx^4} - \alpha_i^4 w(x) = 0 \quad \alpha_i^4 = \frac{\Omega^2 \rho}{EI} \quad (5)$$

and using proper boundary conditions such as

$$\begin{aligned} x = 0 & : & w = 0, & \quad \frac{dw}{dx} = 0 \\ x = L & : & \frac{d^2 w}{dx^2} = 0, & \quad \frac{d^3 w}{dx^3} = 0 \end{aligned}$$

After some calculation, one gets the formula to Elasticity and resonance frequency.

$$E_i = \Omega_i^2 \frac{\rho L^4}{\alpha_i^4 I} \quad (6)$$

where

- E = Young's modulus
- I = area moment of inertia of beam about neutral axis
- L = span of beam
- ρ = mass per unit length
- α_i = 1.87510407
4.69409113
10.99554073 for clamped-free condition $i = 1, 2, 3$

4 Identification Results

FRF's are acquired by 100 averages for each sample to reduce the noise effect. These results are made by using (3) and (6).

$$\zeta_i = \frac{\omega_{i2} - \omega_{i1}}{2\Omega_i}, \quad 0 < \zeta_i$$

$$E_i = \Omega_i^2 \frac{\rho L^4}{\alpha_i^4 I}$$

If the dynamic system has large nonlinearity, then these calculation formulas might not be correct.

LCP-02317						
No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	1.92E+02	4.22E+10	1.40E-01	2.08E+02	4.96E+10	1.24E-01
	1.30E+03	4.90E+10	5.04E-02	1.30E+03	4.90E+10	4.03E-02
	3.49E+03	4.53E+10	2.38E-02	3.47E+03	4.48E+10	2.67E-02
2	2.24E+02	5.75E+10	2.62E-02	1.76E+02	3.55E+10	5.69E-02
	1.31E+03	5.02E+10	4.35E-02	1.28E+03	4.78E+10	5.05E-02
	3.55E+03	4.69E+10	2.22E-02	3.52E+03	4.61E+10	2.56E-02
3	2.08E+02	4.96E+10	1.17E-01	2.08E+02	4.96E+10	1.14E-01
	1.30E+03	4.90E+10	4.51E-02	1.30E+03	4.90E+10	4.69E-02
	3.49E+03	4.53E+10	2.88E-02	3.50E+03	4.57E+10	2.89E-02
4	1.76E+02	3.55E+10	1.22E-01	1.76E+02	3.55E+10	4.71E-02
	1.26E+03	4.66E+10	4.02E-02	1.25E+03	4.54E+10	4.29E-02
	3.33E+03	4.12E+10	2.76E-02	3.33E+03	4.12E+10	2.65E-02
5	1.76E+02	3.55E+10	4.91E-02	1.76E+02	3.55E+10	4.97E-02
	1.23E+03	4.43E+10	3.96E-02	1.23E+03	4.43E+10	3.62E-02
	3.33E+03	4.12E+10	2.91E-02	3.33E+03	4.12E+10	3.01E-02
6	1.76E+02	3.55E+10	4.91E-02	1.76E+02	3.55E+10	6.15E-02
	1.22E+03	4.31E+10	5.04E-02	1.22E+03	4.31E+10	3.68E-02
	3.33E+03	4.12E+10	2.54E-02	3.33E+03	4.12E+10	2.15E-02
7	1.76E+02	3.55E+10	5.69E-02	1.76E+02	3.55E+10	6.02E-02
	1.25E+03	4.54E+10	5.14E-02	1.25E+03	4.54E+10	4.75E-02
	3.33E+03	4.12E+10	3.29E-02	3.36E+03	4.20E+10	2.73E-02
8	2.24E+02	5.75E+10	4.89E-02	2.24E+02	5.75E+10	3.29E-02
	1.28E+03	4.78E+10	4.38E-02	1.31E+03	5.02E+10	3.81E-02
	3.52E+03	4.61E+10	2.20E-02	3.52E+03	4.61E+10	2.67E-02
9	1.76E+02	3.55E+10	8.05E-02	1.76E+02	3.55E+10	6.02E-02
	1.28E+03	4.78E+10	3.81E-02	1.28E+03	4.78E+10	4.73E-02
	3.46E+03	4.44E+10	2.26E-02	3.46E+03	4.44E+10	2.68E-02
10	1.76E+02	3.55E+10	8.51E-02	1.76E+02	3.55E+10	6.81E-02
	1.28E+03	4.78E+10	4.66E-02	1.28E+03	4.78E+10	4.19E-02
	3.46E+03	4.44E+10	2.20E-02	3.49E+03	4.53E+10	2.48E-02

LCP-02318

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.96E+10	1.14E-01	2.08E+02	4.96E+10	1.10E-01
	1.30E+03	4.90E+10	4.03E-02	1.31E+03	5.02E+10	4.29E-02
	3.55E+03	4.69E+10	2.82E-02	3.54E+03	4.65E+10	2.54E-02
2	2.08E+02	4.96E+10	1.23E-01	2.08E+02	4.96E+10	1.29E-01
	1.33E+03	5.14E+10	4.29E-02	1.33E+03	5.14E+10	4.11E-02
	3.62E+03	4.86E+10	2.82E-02	3.60E+03	4.82E+10	3.05E-02
3	1.92E+02	4.22E+10	1.29E-01	2.08E+02	4.96E+10	1.24E-01
	1.30E+03	4.90E+10	3.79E-02	1.30E+03	4.90E+10	4.24E-02
	3.50E+03	4.57E+10	2.71E-02	3.50E+03	4.57E+10	2.66E-02
4	2.08E+02	4.96E+10	1.35E-01	2.08E+02	4.96E+10	1.21E-01
	1.33E+03	5.14E+10	4.27E-02	1.34E+03	5.27E+10	4.19E-02
	3.65E+03	4.95E+10	2.67E-02	3.66E+03	4.99E+10	2.69E-02
5	2.08E+02	4.96E+10	1.12E-01	2.08E+02	4.96E+10	1.18E-01
	1.30E+03	4.90E+10	4.21E-02	1.28E+03	4.78E+10	4.78E-02
	3.46E+03	4.44E+10	2.72E-02	3.47E+03	4.48E+10	2.83E-02
6	2.08E+02	4.96E+10	1.29E-01	1.92E+02	4.22E+10	1.34E-01
	1.28E+03	4.78E+10	4.86E-02	1.28E+03	4.78E+10	4.40E-02
	3.49E+03	4.53E+10	2.73E-02	3.49E+03	4.53E+10	2.73E-02
7	2.08E+02	4.96E+10	1.14E-01	2.08E+02	4.96E+10	1.23E-01
	1.30E+03	4.90E+10	4.45E-02	1.30E+03	4.90E+10	4.16E-02
	3.52E+03	4.61E+10	2.65E-02	3.55E+03	4.69E+10	2.57E-02
8	2.08E+02	4.96E+10	1.39E-01	2.08E+02	4.96E+10	1.09E-01
	1.31E+03	5.02E+10	4.90E-02	1.31E+03	5.02E+10	4.66E-02
	3.58E+03	4.78E+10	2.26E-02	3.58E+03	4.78E+10	2.76E-02
9	2.08E+02	4.96E+10	9.22E-02	2.08E+02	4.96E+10	8.62E-02
	1.34E+03	5.27E+10	5.43E-02	1.33E+03	5.14E+10	5.34E-02
	3.62E+03	4.86E+10	3.04E-02	3.62E+03	4.86E+10	2.85E-02
10	2.24E+02	5.75E+10	9.17E-02	2.24E+02	5.75E+10	9.26E-02
	1.38E+03	5.52E+10	4.32E-02	1.36E+03	5.39E+10	4.32E-02
	3.68E+03	5.04E+10	2.50E-02	3.68E+03	5.04E+10	2.48E-02

LCP-02319

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.99E+10	1.17E-01	1.92E+02	4.25E+10	1.58E-01
	1.26E+03	4.69E+10	4.54E-02	1.28E+03	4.81E+10	4.32E-02
	3.39E+03	4.31E+10	3.06E-02	3.41E+03	4.35E+10	3.00E-02
2	2.08E+02	4.99E+10	1.09E-01	2.08E+02	4.99E+10	1.09E-01
	1.30E+03	4.93E+10	4.35E-02	1.31E+03	5.05E+10	3.88E-02
	3.54E+03	4.68E+10	2.95E-02	3.57E+03	4.76E+10	2.83E-02
3	2.08E+02	4.99E+10	1.12E-01	2.08E+02	4.99E+10	1.10E-01
	1.28E+03	4.81E+10	4.84E-02	1.30E+03	4.93E+10	4.89E-02
	3.58E+03	4.81E+10	2.68E-02	3.58E+03	4.81E+10	3.11E-02
4	2.08E+02	4.99E+10	1.05E-01	2.08E+02	4.99E+10	9.75E-02
	1.26E+03	4.69E+10	4.70E-02	1.26E+03	4.69E+10	5.14E-02
	3.42E+03	4.39E+10	2.92E-02	3.46E+03	4.47E+10	2.89E-02
5	2.08E+02	4.99E+10	1.20E-01	2.08E+02	4.99E+10	1.13E-01
	1.26E+03	4.69E+10	4.47E-02	1.28E+03	4.81E+10	3.69E-02
	3.42E+03	4.39E+10	3.18E-02	3.46E+03	4.47E+10	2.94E-02
6	2.24E+02	5.78E+10	1.01E-01	2.08E+02	4.99E+10	1.19E-01
	1.33E+03	5.17E+10	4.16E-02	1.33E+03	5.17E+10	4.47E-02
	3.58E+03	4.81E+10	3.04E-02	3.58E+03	4.81E+10	3.04E-02
7	1.92E+02	4.25E+10	1.18E-01	1.92E+02	4.25E+10	1.24E-01
	1.22E+03	4.34E+10	5.17E-02	1.22E+03	4.34E+10	4.63E-02
	3.33E+03	4.15E+10	2.86E-02	3.34E+03	4.18E+10	2.80E-02
8	2.08E+02	4.99E+10	1.15E-01	2.08E+02	4.99E+10	1.29E-01
	1.33E+03	5.17E+10	4.61E-02	1.33E+03	5.17E+10	4.29E-02
	3.55E+03	4.72E+10	2.61E-02	3.58E+03	4.81E+10	2.68E-02
9	2.08E+02	4.99E+10	1.06E-01	2.08E+02	4.99E+10	1.21E-01
	1.31E+03	5.05E+10	4.14E-02	1.31E+03	5.05E+10	4.66E-02
	3.55E+03	4.72E+10	2.70E-02	3.58E+03	4.81E+10	2.57E-02
10	2.08E+02	4.99E+10	1.07E-01	2.08E+02	4.99E+10	1.25E-01
	1.31E+03	5.05E+10	5.24E-02	1.33E+03	5.17E+10	5.31E-02
	3.54E+03	4.68E+10	2.87E-02	3.58E+03	4.81E+10	2.90E-02

LCP-02320

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.24E+02	5.75E+10	1.09E-01	2.24E+02	5.75E+10	1.16E-01
	1.39E+03	5.65E+10	4.27E-02	1.39E+03	5.65E+10	4.27E-02
	3.74E+03	5.21E+10	2.59E-02	3.78E+03	5.30E+10	2.76E-02
2	2.08E+02	4.96E+10	1.40E-01	2.24E+02	5.75E+10	1.02E-01
	1.36E+03	5.39E+10	4.03E-02	1.38E+03	5.52E+10	4.12E-02
	3.66E+03	4.99E+10	2.41E-02	3.66E+03	4.99E+10	2.53E-02
3	2.24E+02	5.75E+10	1.10E-01	2.24E+02	5.75E+10	1.15E-01
	1.36E+03	5.39E+10	4.85E-02	1.34E+03	5.27E+10	4.53E-02
	3.62E+03	4.86E+10	2.42E-02	3.62E+03	4.86E+10	2.77E-02
4	2.08E+02	4.96E+10	1.21E-01	2.24E+02	5.75E+10	1.11E-01
	1.36E+03	5.39E+10	4.40E-02	1.36E+03	5.39E+10	3.96E-02
	3.58E+03	4.78E+10	2.68E-02	3.62E+03	4.86E+10	2.60E-02
5	2.24E+02	5.75E+10	1.11E-01	2.24E+02	5.75E+10	1.06E-01
	1.38E+03	5.52E+10	4.19E-02	1.38E+03	5.52E+10	4.42E-02
	3.68E+03	5.04E+10	2.54E-02	3.71E+03	5.13E+10	2.77E-02
6	2.24E+02	5.75E+10	1.06E-01	2.24E+02	5.75E+10	1.19E-01
	1.38E+03	5.52E+10	3.75E-02	1.38E+03	5.52E+10	4.35E-02
	3.73E+03	5.17E+10	2.93E-02	3.71E+03	5.13E+10	2.66E-02
7	2.24E+02	5.75E+10	9.82E-02	2.24E+02	5.75E+10	1.13E-01
	1.41E+03	5.78E+10	4.09E-02	1.41E+03	5.78E+10	4.35E-02
	3.74E+03	5.21E+10	2.46E-02	3.74E+03	5.21E+10	2.40E-02
8	2.24E+02	5.75E+10	9.77E-02	2.08E+02	4.96E+10	1.31E-01
	1.42E+03	5.91E+10	3.98E-02	1.42E+03	5.91E+10	3.88E-02
	3.74E+03	5.21E+10	2.55E-02	3.81E+03	5.39E+10	3.00E-02
9	2.08E+02	4.96E+10	1.32E-01	2.24E+02	5.75E+10	1.15E-01
	1.41E+03	5.78E+10	3.89E-02	1.41E+03	5.78E+10	3.76E-02
	3.71E+03	5.13E+10	2.74E-02	3.74E+03	5.21E+10	2.22E-02
10	2.24E+02	5.75E+10	9.93E-02	2.24E+02	5.75E+10	9.21E-02
	1.41E+03	5.78E+10	4.91E-02	1.41E+03	5.78E+10	4.75E-02
	3.78E+03	5.30E+10	2.29E-02	3.78E+03	5.30E+10	2.36E-02

O-02329

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	1.76E+02	3.36E+10	1.81E-01	1.76E+02	3.36E+10	1.85E-01
	1.25E+03	4.31E+10	3.46E-02	1.26E+03	4.42E+10	3.04E-02
	3.39E+03	4.06E+10	9.93E-03	3.39E+03	4.06E+10	1.19E-02
2	1.76E+02	3.36E+10	1.85E-01	2.08E+02	4.70E+10	1.73E-01
	1.25E+03	4.31E+10	4.40E-02	1.28E+03	4.53E+10	3.27E-02
	3.46E+03	4.21E+10	1.02E-02	3.46E+03	4.21E+10	1.12E-02
3	1.76E+02	3.36E+10	1.81E-01	1.76E+02	3.36E+10	1.74E-01
	1.26E+03	4.42E+10	4.48E-02	1.26E+03	4.42E+10	5.00E-02
	3.39E+03	4.06E+10	1.07E-02	3.39E+03	4.06E+10	9.27E-03
4	1.76E+02	3.36E+10	1.78E-01	1.76E+02	3.36E+10	1.58E-01
	1.26E+03	4.42E+10	4.70E-02	1.26E+03	4.42E+10	5.09E-02
	3.42E+03	4.13E+10	1.01E-02	3.41E+03	4.10E+10	8.82E-03
5	1.76E+02	3.36E+10	1.83E-01	1.76E+02	3.36E+10	1.92E-01
	1.22E+03	4.09E+10	5.37E-02	1.22E+03	4.09E+10	5.12E-02
	3.33E+03	3.91E+10	7.69E-03	3.33E+03	3.91E+10	7.06E-03
6	1.76E+02	3.36E+10	1.87E-01	1.76E+02	3.36E+10	1.81E-01
	1.22E+03	4.09E+10	4.72E-02	1.22E+03	4.09E+10	4.55E-02
	3.31E+03	3.87E+10	1.09E-02	3.30E+03	3.83E+10	1.25E-02
7	1.92E+02	4.00E+10	1.85E-01	2.08E+02	4.70E+10	1.44E-01
	1.28E+03	4.53E+10	3.70E-02	1.28E+03	4.53E+10	4.05E-02
	3.44E+03	4.17E+10	9.61E-03	3.46E+03	4.21E+10	1.02E-02
8	1.76E+02	3.36E+10	1.83E-01	1.76E+02	3.36E+10	1.62E-01
	1.22E+03	4.09E+10	3.84E-02	1.22E+03	4.09E+10	4.32E-02
	3.31E+03	3.87E+10	9.01E-03	3.33E+03	3.91E+10	9.69E-03
9	1.76E+02	3.36E+10	1.74E-01	1.76E+02	3.36E+10	1.80E-01
	1.18E+03	3.88E+10	5.84E-02	1.22E+03	4.09E+10	5.06E-02
	3.33E+03	3.91E+10	1.04E-02	3.33E+03	3.91E+10	1.16E-02
10	1.76E+02	3.36E+10	1.78E-01	1.76E+02	3.36E+10	1.67E-01
	1.28E+03	4.53E+10	5.48E-02	1.26E+03	4.42E+10	5.41E-02
	3.41E+03	4.10E+10	7.81E-03	3.39E+03	4.06E+10	1.09E-02

O-02330

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.24E+02	5.92E+10	1.21E-01	2.08E+02	5.11E+10	1.24E-01
	1.41E+03	5.96E+10	4.20E-02	1.42E+03	6.09E+10	2.65E-02
	3.78E+03	5.47E+10	1.40E-02	3.78E+03	5.47E+10	1.43E-02
2	2.08E+02	5.11E+10	1.47E-01	2.08E+02	5.11E+10	1.40E-01
	1.41E+03	5.96E+10	2.80E-02	1.39E+03	5.82E+10	2.88E-02
	3.74E+03	5.37E+10	8.99E-03	3.74E+03	5.37E+10	9.85E-03
3	2.24E+02	5.92E+10	1.32E-01	2.24E+02	5.92E+10	1.29E-01
	1.38E+03	5.69E+10	3.22E-02	1.38E+03	5.69E+10	3.09E-02
	3.71E+03	5.28E+10	8.69E-03	3.71E+03	5.28E+10	9.56E-03
4	2.24E+02	5.92E+10	1.31E-01	2.08E+02	5.11E+10	1.60E-01
	1.39E+03	5.82E+10	3.55E-02	1.39E+03	5.82E+10	4.02E-02
	3.74E+03	5.37E+10	8.37E-03	3.71E+03	5.28E+10	1.02E-02
5	2.08E+02	5.11E+10	1.58E-01	2.24E+02	5.92E+10	1.37E-01
	1.41E+03	5.96E+10	2.77E-02	1.42E+03	6.09E+10	3.69E-02
	3.74E+03	5.37E+10	1.07E-02	3.74E+03	5.37E+10	1.08E-02
6	2.08E+02	5.11E+10	1.56E-01	2.08E+02	5.11E+10	1.51E-01
	1.38E+03	5.69E+10	3.82E-02	1.38E+03	5.69E+10	3.16E-02
	3.71E+03	5.28E+10	1.15E-02	3.71E+03	5.28E+10	1.27E-02
7	2.08E+02	5.11E+10	1.29E-01	2.08E+02	5.11E+10	1.83E-01
	1.34E+03	5.43E+10	3.65E-02	1.36E+03	5.56E+10	2.95E-02
	3.65E+03	5.10E+10	1.15E-02	3.63E+03	5.06E+10	8.88E-03
8	2.08E+02	5.11E+10	1.79E-01	2.08E+02	5.11E+10	1.25E-01
	1.39E+03	5.82E+10	2.95E-02	1.38E+03	5.69E+10	3.44E-02
	3.68E+03	5.19E+10	9.96E-03	3.68E+03	5.19E+10	1.16E-02
9	2.08E+02	5.11E+10	1.81E-01	2.08E+02	5.11E+10	1.44E-01
	1.34E+03	5.43E+10	3.32E-02	1.36E+03	5.56E+10	3.66E-02
	3.68E+03	5.19E+10	1.49E-02	3.68E+03	5.19E+10	8.08E-03
10	2.08E+02	5.11E+10	1.54E-01	2.08E+02	5.11E+10	1.54E-01
	1.34E+03	5.43E+10	3.99E-02	1.34E+03	5.43E+10	3.70E-02
	3.58E+03	4.92E+10	1.02E-02	3.55E+03	4.84E+10	1.17E-02

O-02331

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.82E+10	1.18E-01	2.08E+02	4.82E+10	1.34E-01
	1.25E+03	4.42E+10	4.65E-02	1.23E+03	4.31E+10	3.96E-02
	3.36E+03	4.08E+10	9.26E-03	3.36E+03	4.08E+10	8.76E-03
2	2.08E+02	4.82E+10	9.31E-02	2.08E+02	4.82E+10	8.25E-02
	1.31E+03	4.88E+10	2.77E-02	1.28E+03	4.65E+10	2.84E-02
	3.47E+03	4.36E+10	7.76E-03	3.49E+03	4.40E+10	8.82E-03
3	2.08E+02	4.82E+10	1.24E-01	2.08E+02	4.82E+10	1.29E-01
	1.30E+03	4.76E+10	3.72E-02	1.30E+03	4.76E+10	3.60E-02
	3.46E+03	4.32E+10	8.70E-03	3.46E+03	4.32E+10	9.90E-03
4	2.08E+02	4.82E+10	1.34E-01	2.08E+02	4.82E+10	1.32E-01
	1.28E+03	4.65E+10	3.97E-02	1.28E+03	4.65E+10	3.12E-02
	3.49E+03	4.40E+10	8.62E-03	3.49E+03	4.40E+10	9.22E-03
5	2.08E+02	4.82E+10	1.36E-01	2.08E+02	4.82E+10	1.34E-01
	1.28E+03	4.65E+10	3.32E-02	1.28E+03	4.65E+10	3.36E-02
	3.41E+03	4.20E+10	9.03E-03	3.39E+03	4.16E+10	9.68E-03
6	2.08E+02	4.82E+10	1.27E-01	2.08E+02	4.82E+10	1.46E-01
	1.28E+03	4.65E+10	3.48E-02	1.28E+03	4.65E+10	2.35E-02
	3.46E+03	4.32E+10	7.40E-03	3.46E+03	4.32E+10	7.40E-03
7	2.08E+02	4.82E+10	1.30E-01	2.08E+02	4.82E+10	1.37E-01
	1.28E+03	4.65E+10	4.58E-02	1.28E+03	4.65E+10	3.81E-02
	3.47E+03	4.36E+10	9.46E-03	3.46E+03	4.32E+10	9.30E-03
8	2.08E+02	4.82E+10	1.26E-01	2.08E+02	4.82E+10	1.30E-01
	1.25E+03	4.42E+10	3.49E-02	1.26E+03	4.53E+10	3.16E-02
	3.39E+03	4.16E+10	7.64E-03	3.39E+03	4.16E+10	9.48E-03
9	2.08E+02	4.82E+10	1.31E-01	2.08E+02	4.82E+10	1.32E-01
	1.26E+03	4.53E+10	3.57E-02	1.26E+03	4.53E+10	3.49E-02
	3.41E+03	4.20E+10	1.02E-02	3.42E+03	4.24E+10	9.99E-03
10	2.08E+02	4.82E+10	9.25E-02	2.08E+02	4.82E+10	1.30E-01
	1.26E+03	4.53E+10	3.94E-02	1.25E+03	4.42E+10	6.77E-02
	3.39E+03	4.16E+10	9.07E-03	3.39E+03	4.16E+10	7.44E-03

O-02332

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.24E+02	5.92E+10	1.24E-01	2.24E+02	5.92E+10	1.21E-01
	1.39E+03	5.82E+10	3.35E-02	1.41E+03	5.96E+10	3.39E-02
	3.78E+03	5.47E+10	1.28E-02	3.76E+03	5.42E+10	1.15E-02
2	2.24E+02	5.92E+10	1.14E-01	2.08E+02	5.11E+10	1.45E-01
	1.41E+03	5.96E+10	3.83E-02	1.41E+03	5.96E+10	3.12E-02
	3.76E+03	5.42E+10	1.15E-02	3.78E+03	5.47E+10	1.14E-02
3	2.24E+02	5.92E+10	1.25E-01	2.24E+02	5.92E+10	1.21E-01
	1.38E+03	5.69E+10	4.14E-02	1.39E+03	5.82E+10	3.80E-02
	3.73E+03	5.33E+10	1.38E-02	3.73E+03	5.33E+10	1.30E-02
4	2.08E+02	5.11E+10	1.24E-01	2.24E+02	5.92E+10	1.17E-01
	1.41E+03	5.96E+10	4.05E-02	1.41E+03	5.96E+10	3.09E-02
	3.74E+03	5.37E+10	1.19E-02	3.76E+03	5.42E+10	1.34E-02
5	2.24E+02	5.92E+10	1.14E-01	2.24E+02	5.92E+10	1.30E-01
	1.41E+03	5.96E+10	3.68E-02	1.41E+03	5.96E+10	3.04E-02
	3.78E+03	5.47E+10	1.16E-02	3.78E+03	5.47E+10	1.42E-02
6	2.24E+02	5.92E+10	1.15E-01	2.08E+02	5.11E+10	1.28E-01
	1.41E+03	5.96E+10	2.74E-02	1.41E+03	5.96E+10	3.57E-02
	3.74E+03	5.37E+10	1.34E-02	3.74E+03	5.37E+10	1.40E-02
7	2.24E+02	5.92E+10	1.27E-01	2.24E+02	5.92E+10	1.25E-01
	1.39E+03	5.82E+10	3.05E-02	1.39E+03	5.82E+10	3.16E-02
	3.71E+03	5.28E+10	1.39E-02	3.73E+03	5.33E+10	1.14E-02
8	2.24E+02	5.92E+10	1.30E-01	2.24E+02	5.92E+10	1.18E-01
	1.41E+03	5.96E+10	2.22E-02	1.41E+03	5.96E+10	3.47E-02
	3.76E+03	5.42E+10	1.18E-02	3.76E+03	5.42E+10	1.29E-02
9	2.08E+02	5.11E+10	1.26E-01	2.08E+02	5.11E+10	1.25E-01
	1.39E+03	5.82E+10	3.12E-02	1.39E+03	5.82E+10	2.56E-02
	3.76E+03	5.42E+10	1.31E-02	3.74E+03	5.37E+10	1.28E-02
10	2.24E+02	5.92E+10	1.35E-01	2.24E+02	5.92E+10	1.27E-01
	1.39E+03	5.82E+10	3.72E-02	1.39E+03	5.82E+10	2.85E-02
	3.74E+03	5.37E+10	1.34E-02	3.73E+03	5.33E+10	1.24E-02

For RX-U-37, RX-U-38, and RX-U-39 :

U-37

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.80E+10	1.23E-01	2.08E+02	4.80E+10	1.26E-01
	1.34E+03	5.11E+10	4.39E-02	1.33E+03	4.99E+10	3.37E-02
	3.65E+03	4.80E+10	1.38E-02	3.62E+03	4.72E+10	1.43E-02
2	2.08E+02	4.80E+10	1.26E-01	2.08E+02	4.80E+10	1.27E-01
	1.33E+03	4.99E+10	3.37E-02	1.33E+03	4.99E+10	3.54E-02
	3.62E+03	4.72E+10	1.43E-02	3.62E+03	4.72E+10	1.35E-02
3	2.08E+02	4.80E+10	1.17E-01	2.08E+02	4.80E+10	1.26E-01
	1.31E+03	4.87E+10	3.65E-02	1.31E+03	4.87E+10	3.08E-02
	3.57E+03	4.59E+10	1.22E-02	3.55E+03	4.55E+10	1.52E-02

U-38

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.64E+10	1.16E-01	1.92E+02	3.95E+10	1.34E-01
	1.28E+03	4.47E+10	3.74E-02	1.28E+03	4.47E+10	3.35E-02
	3.46E+03	4.16E+10	1.27E-02	3.49E+03	4.24E+10	1.27E-02
2	1.92E+02	3.95E+10	1.34E-01	2.08E+02	4.64E+10	1.19E-01
	1.28E+03	4.47E+10	3.35E-02	1.30E+03	4.58E+10	3.47E-02
	3.49E+03	4.24E+10	1.27E-02	3.49E+03	4.24E+10	1.50E-02
3	1.92E+02	3.95E+10	1.42E-01	2.08E+02	4.64E+10	1.09E-01
	1.26E+03	4.36E+10	3.49E-02	1.28E+03	4.47E+10	3.87E-02

U-39

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.82E+10	1.26E-01	2.24E+02	5.59E+10	1.10E-01
	1.36E+03	5.25E+10	3.16E-02	1.38E+03	5.37E+10	2.97E-02
	3.66E+03	4.86E+10	1.40E-02	3.71E+03	4.98E+10	1.41E-02
2	2.24E+02	5.59E+10	1.10E-01	2.24E+02	5.59E+10	1.26E-01
	1.38E+03	5.37E+10	2.97E-02	1.38E+03	5.37E+10	3.36E-02
	3.71E+03	4.98E+10	1.41E-02	3.71E+03	4.98E+10	1.34E-02
3	2.08E+02	4.82E+10	1.35E-01	2.08E+02	4.82E+10	1.29E-01
	1.38E+03	5.37E+10	3.72E-02	1.38E+03	5.37E+10	3.01E-02
	3.68E+03	4.90E+10	1.28E-02	3.68E+03	4.90E+10	1.25E-02

For RX-L-21, RX-L-23, and RX-L-max :

L-21

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	1.92E+02	4.03E+10	1.28E-01	2.08E+02	4.73E+10	1.15E-01
	1.28E+03	4.56E+10	5.42E-02	1.26E+03	4.45E+10	5.21E-02
	3.42E+03	4.16E+10	1.48E-02	3.42E+03	4.16E+10	1.18E-02
2	2.08E+02	4.73E+10	1.15E-01	1.92E+02	4.03E+10	1.34E-01
	1.26E+03	4.45E+10	5.21E-02	1.28E+03	4.56E+10	5.58E-02
	3.42E+03	4.16E+10	1.18E-02	3.44E+03	4.20E+10	1.43E-02
3	2.08E+02	4.73E+10	1.23E-01	2.08E+02	4.73E+10	1.20E-01
	1.25E+03	4.33E+10	3.30E-02	1.26E+03	4.45E+10	3.86E-02
	3.39E+03	4.08E+10	1.25E-02	3.38E+03	4.05E+10	1.44E-02

L-23

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.74E+10	1.24E-01	2.08E+02	4.74E+10	1.28E-01
	1.30E+03	4.69E+10	3.15E-02	1.30E+03	4.69E+10	3.41E-02
	3.46E+03	4.25E+10	1.46E-02	3.44E+03	4.21E+10	1.28E-02
2	2.08E+02	4.74E+10	1.28E-01	2.08E+02	4.74E+10	1.20E-01
	1.30E+03	4.69E+10	3.41E-02	1.30E+03	4.69E+10	3.23E-02
	3.44E+03	4.21E+10	1.28E-02	3.44E+03	4.21E+10	1.20E-02
3	2.08E+02	4.74E+10	1.24E-01	2.08E+02	4.74E+10	1.16E-01
	1.30E+03	4.69E+10	3.33E-02	1.30E+03	4.69E+10	3.81E-02
	3.46E+03	4.25E+10	1.12E-02	3.47E+03	4.29E+10	1.48E-02

L-max

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.83E+10	1.35E-01	2.08E+02	4.83E+10	1.36E-01
	1.34E+03	5.14E+10	3.48E-02	1.33E+03	5.02E+10	3.81E-02
	3.63E+03	4.79E+10	1.22E-02	3.55E+03	4.58E+10	1.17E-02
2	2.08E+02	4.83E+10	1.36E-01	2.08E+02	4.83E+10	1.24E-01
	1.33E+03	5.02E+10	3.81E-02	1.30E+03	4.78E+10	4.02E-02
	3.55E+03	4.58E+10	1.17E-02	3.57E+03	4.62E+10	1.49E-02

For RX-COC-25, RX-COC-26, RX-COC-27, and RX-COC-28 :

COC-25

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	1.92E+02	3.45E+10	1.36E-01	2.08E+02	4.05E+10	1.20E-01
	1.18E+03	3.34E+10	4.06E-02	1.18E+03	3.34E+10	4.40E-02
	3.17E+03	3.05E+10	1.11E-02	3.15E+03	3.02E+10	1.51E-02
2	2.08E+02	4.05E+10	1.20E-01	1.92E+02	3.45E+10	1.17E-01
	1.18E+03	3.34E+10	4.40E-02	1.17E+03	3.25E+10	3.55E-02
	3.15E+03	3.02E+10	1.51E-02	3.15E+03	3.02E+10	1.41E-02
3	1.92E+02	3.45E+10	1.33E-01	2.08E+02	4.05E+10	1.31E-01
	1.18E+03	3.34E+10	3.75E-02	1.18E+03	3.34E+10	4.80E-02
	3.26E+03	3.24E+10	1.33E-02	3.26E+03	3.24E+10	1.03E-02

COC-26

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.40E+10	1.27E-01	2.08E+02	4.40E+10	1.23E-01
	1.31E+03	4.45E+10	2.77E-02	1.31E+03	4.45E+10	4.46E-02
	3.52E+03	4.09E+10	1.08E-02	3.54E+03	4.13E+10	1.24E-02
2	2.08E+02	4.40E+10	1.23E-01	2.24E+02	5.10E+10	1.21E-01
	1.31E+03	4.45E+10	4.46E-02	1.31E+03	4.45E+10	3.75E-02
	3.54E+03	4.13E+10	1.24E-02	3.54E+03	4.13E+10	1.11E-02
3	2.08E+02	4.40E+10	1.30E-01	2.08E+02	4.40E+10	1.29E-01
	1.33E+03	4.56E+10	2.92E-02	1.33E+03	4.56E+10	3.23E-02
	3.55E+03	4.16E+10	1.20E-02	3.57E+03	4.20E+10	1.28E-02

COC-27

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	1.92E+02	3.46E+10	1.37E-01	2.08E+02	4.06E+10	1.21E-01
	1.25E+03	3.73E+10	3.28E-02	1.25E+03	3.73E+10	3.51E-02
	3.39E+03	3.51E+10	1.60E-02	3.36E+03	3.44E+10	1.31E-02
2	2.08E+02	4.06E+10	1.21E-01	1.92E+02	3.46E+10	1.38E-01
	1.25E+03	3.73E+10	3.51E-02	1.25E+03	3.73E+10	3.44E-02
	3.36E+03	3.44E+10	1.31E-02	3.36E+03	3.44E+10	1.07E-02
3	1.92E+02	3.46E+10	1.38E-01	2.08E+02	4.06E+10	1.29E-01
	1.23E+03	3.63E+10	3.39E-02	1.23E+03	3.63E+10	3.13E-02
	3.39E+03	3.51E+10	1.20E-02	3.39E+03	3.51E+10	1.23E-02

COC-28

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	2.08E+02	4.38E+10	1.35E-01	2.08E+02	4.38E+10	1.25E-01
	1.31E+03	4.44E+10	3.56E-02	1.34E+03	4.66E+10	3.14E-02
	3.52E+03	4.08E+10	1.28E-02	3.55E+03	4.15E+10	1.08E-02
2	2.08E+02	4.38E+10	1.25E-01	2.24E+02	5.08E+10	1.20E-01
	1.34E+03	4.66E+10	3.14E-02	1.34E+03	4.66E+10	3.93E-02
	3.55E+03	4.15E+10	1.08E-02	3.57E+03	4.19E+10	1.36E-02
3	2.08E+02	4.38E+10	1.14E-01	2.08E+02	4.38E+10	1.25E-01
	1.33E+03	4.55E+10	3.64E-02	1.33E+03	4.55E+10	3.15E-02
	3.58E+03	4.22E+10	1.15E-02	3.58E+03	4.22E+10	1.17E-02

For aluminum material (actually the density of this material is slightly less than that in material table) :

Aluminum

No.	Resonance	Elasticity[Pa]	Damping	Resonance	Elasticity[Pa]	Damping
1	1.60E+02	4.02E+10	1.74E-01	1.76E+02	4.87E+10	1.55E-01
	9.60E+02	3.69E+10	5.54E-02	9.44E+02	3.57E+10	5.57E-02
	2.86E+03	4.19E+10	1.06E-02	2.86E+03	4.19E+10	1.12E-02
	5.63E+03	4.22E+10	5.41E-03	5.63E+03	4.22E+10	5.64E-03
2	1.76E+02	4.87E+10	1.55E-01	1.76E+02	4.87E+10	1.73E-01
	9.44E+02	3.57E+10	5.57E-02	9.60E+02	3.69E+10	4.79E-02
	2.86E+03	4.19E+10	1.12E-02	2.86E+03	4.19E+10	1.18E-02
	5.63E+03	4.22E+10	5.64E-03	5.63E+03	4.22E+10	5.47E-03

note : These results are attached as an Excel file(result1.xls).